Exploring the spectrum of QCD using a space-time lattice

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spectroscopy is a powerful tool for distilling key degrees of freedom

calculating spectrum of QCD → introduction of space-time lattice
  - spectrum determination requires extraction of excited-state energies
  - discuss how to extract excited-state energies from Monte Carlo estimates of correlation functions in Euclidean lattice field theory

past explorations:
  - Yang-Mills glueballs
  - heavy-quark hybrid mesons

current exploration:
  - baryon and meson spectrum (progress report)
Monte Carlo method with space-time lattice

- Introduction of space-time lattice allows Monte Carlo evaluation of path integrals needed to extract spectrum from QCD Lagrangian

\[ L_{QCD} \]
Lagrangian of QCD

- Tool to search for better ways of calculating in gauge theories
  - What dominates the path integrals? (instantons, center vortices, …)
  - Construction of effective field theory of glue? (strings, …)

hadron spectrum, structure, transitions
Energies from correlation functions

- stationary state energies can be extracted from asymptotic decay rate of temporal correlations of the fields (in the imaginary time formalism)
- evolution in Heisenberg picture \( \phi(t) = e^{Ht} \phi(0) e^{-Ht} \) \((H = \text{Hamiltonian})\)
- spectral representation of a simple correlation function
  - assume transfer matrix, ignore temporal boundary conditions
  - focus only on one time ordering
    \[
    \langle 0 | \phi(t) \phi(0) | 0 \rangle = \sum_n \langle 0 | e^{Ht} \phi(0) e^{-Ht} | n \rangle \langle n | \phi(0) | 0 \rangle
    \]
    \[
    = \sum_n |\langle n | \phi(0) | 0 \rangle|^2 e^{-(E_n - E_0)t} = \sum_n A_n e^{-(E_n - E_0)t}
    \]
  - extract \( A_1 \) and \( E_1 - E_0 \) as \( t \to \infty \)

(assuming \( \langle 0 | \phi(0) | 0 \rangle = 0 \) and \( \langle 1 | \phi(0) | 0 \rangle \neq 0 \))
Quantum mechanics and path integrals

- in the 1940’s, Feynman developed an alternative formulation of quantum mechanics (his Ph.D. thesis)
  - Richard Feynman, Rev Mod Phys 20, 367 (1948)
- quantum mechanical law of motion:
  - probability amplitude from *sum over histories*

\[ Z(b,a) \sim \sum_{\text{all paths}} \exp\left(\frac{iS}{\hbar}\right) \]

- all paths contribute to the probability amplitude, but with different *phases* determined by the *action* \( S \)
- classical limit: when small changes in path yield changes in action large compared to \( \hbar \), phases cancel out and path of least action \( \delta S = 0 \) dominates sum over histories
Monte Carlo Evaluation of Path Integrals

- vacuum expectation value in terms of path integrals
  \[
  \langle \Phi(t)\Phi^*(0) \rangle = \frac{\int D\Phi \Phi(t)\Phi^*(0) e^{-S[\Phi]}}{\int D\Phi e^{-S[\Phi]}}
  \]

- \(S[\Phi]\) is the Euclidean space action, \(\Phi^*(t)\) creates state of interest
- path integral weight is real and positive \(\Rightarrow\) probability interpretation
- Markov-chain Monte Carlo methods can be applied
  - Metropolis, heatbath, overrelaxation
  - hybrid methods
  - no expansions in a small parameter
  - statistical errors
- first principles approach
Need for Monte Carlo Method in QCD

- calculational tools of QED (small coupling expansions)
  - work well for deep inelastic scattering in QCD
    - asymptotic freedom
  - utterly fail for hadron formation
    - bound state problem \( \propto e^{-1/g^2} \)

  - formulate QCD using a discrete space-time lattice

- advantage of this approach
  - facilitates computer simulations of quarks and gluons
  - theorists now free of the shackles of small-coupling expansions!
Birth of Lattice QCD


Confinement of quarks

Kenneth G. Wilson

A mechanism for the confinement of quarks, similar to that of Schwinger, is defined which requires
the existence of non-Abelian gauge fields. It is shown that such a gauge field theory
on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and tracing the
gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has
a computable strong-coupling limit, in this limit the binding mechanism applies and there are no free
quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The
strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice)
yield quark paths. This structure is reminiscent of relativistic string models of hadrons.

I. INTRODUCTION

The success of the quark-parton picture both for resonances and for deep-inelastic electron and neutrino processes makes it difficult to believe quarks do not exist. The problem is that quarks have not been seen. This suggests that quark, for some reason, cannot appear as separate particles in a final state. A number of speculations have been offered as to how this might happen.

Independently of the quark problem, Schwinger observed many years ago that the vector mesons of a gauge theory can have a nonzero mass if vacuum polarization totally screens the charges in a gauge theory. Schwinger illustrated this result with the exact solution of quantum electrodynamics in one space and one time dimension, where the photon acquires a mass $-e^2$ for any nonzero charge $e$ (has dimensions of mass $^{1/2}$ in this theory). Schwinger suggested that the same effect could occur in four dimensions for sufficiently large couplings.

Further study of the Schwinger model by Lowenstein and Swieca and Casher, Kogut, and Lassik has shown that the asymptotic states of the model contain only massive photons, not electrons. Nevertheless, as Chabot et al., have shown in detail, the electrons are present in deep-inelastic processes and behave like free pointlike particles over short times and short distances. The polarization effects which prevent the appearance of electrons in the final state take place on a longer time scale (longer than $1/\alpha$, where $\alpha$ is the photon mass).

A new mechanism which keeps quarks bound will be proposed in this paper. The mechanism applies to gauge theories only. The mechanism will be illustrated using the strong-coupling limit of a gauge theory in four-dimensional space-time. However, the model discussed here has a built-in ultraviolet cutoff, and in the strong-coupling limit all particle masses (including the gauge field masses) are much larger than the cutoff, in consequence the theory is far from covariant.

The confinement mechanism proposed here is soft (long-time scale). However, in the model discussed here the cutoff spoils the possibility of free pointlike behavior for the quarks.

The model discussed in this paper is a gauge theory set up on a four-dimensional Euclidean lattice. The inverse of the lattice spacing serves as an ultraviolet cutoff, because of a Euclidean space (i.e., imaginary instead of real times) instead of a Lorentz space is not a serious restriction; the energy eigenstates (including scattering states) of the lattice theory can be determined from the "transfer-matrix" formalism as has been discussed by Wilcox and Wilson.

Ken Wilson

November 9, 2005 Exploring spectrum (C. Morningstar)
Lattice QCD

- hypercubic space-time lattice
- quarks reside on sites, gluons reside on links between sites
- systematic errors
  - discretization (physics $\rightarrow$ continuum limit)
  - finite volume
- simulation via computer
- use of Monte Carlo methods

quarks

 gluons
Effective mass

- the “effective mass” is given by $m_{\text{eff}}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right)$
- notice that (take $E_0 = 0$)
\[
\lim_{t \to \infty} m_{\text{eff}}(t) = \ln\left(\frac{A_1 e^{-E_1t} + A_2 e^{-E_2t} + \cdots}{A_1 e^{-E_1(t+1)} + \cdots}\right) \to \ln e^{-E_1} = E_1
\]
- the effective mass tends to the actual mass (energy) asymptotically
- effective mass plot is convenient visual tool to see signal extraction
  - seen as a plateau
- plateau sets in quickly for good operator
- excited-state contamination before plateau
Reducing contamination

- statistical noise generally increases with temporal separation $t$
- effective masses associated with correlation functions of simple local fields do not reach a plateau before noise swamps the signal
  - need better operators
  - better operators have reduced couplings with higher-lying contaminating states
- recipe for making better operators
  - crucial to construct operators using *smeared* fields
    - link variable smearing
    - quark field smearing
  - spatially extended operators
  - use large *set* of operators (variational coefficients)
Principal correlators

- extracting excited-state energies described in
  - C. Michael, NPB 259, 58 (1985)
  - Luscher and Wolff, NPB 339, 222 (1990)
- can be viewed as exploiting the variational method
- for a given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle 0 | O_{\alpha}(t) O_{\beta}^+(0) | 0 \rangle$ one defines the $N$ principal correlators $\lambda_{\alpha}(t, t_0)$ as the eigenvalues of
  
  \[ C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} \]

  where $t_0$ (the time defining the “metric”) is small
- can show that $\lim_{t \to \infty} \lambda_{\alpha}(t, t_0) = e^{-(t-t_0)E_\alpha} (1 + e^{-t\Delta E_\alpha})$
- $N$ principal effective masses defined by $m_{\alpha}^{\text{eff}}(t) = \ln \left( \frac{\lambda_{\alpha}(t, t_0)}{\lambda_{\alpha}(t+1, t_0)} \right)$ now tend (plateau) to the $N$ lowest-lying stationary-state energies
Principal effective masses

- just need to perform single-exponential fit to each principal correlator to extract spectrum!
  - can again use sum of two-exponentials to minimize sensitivity to $t_{\text{min}}$
- note that principal effective masses (as functions of time) can cross, approach asymptotic behavior from below
- final results are independent of $t_0$, but choosing larger values of this reference time can introduce larger errors
Unstable particles (resonances)

- our computations done in a periodic box
  - momenta quantized
  - discrete energy spectrum of stationary states $\rightarrow$ single hadron, 2 hadron, …

- scattering phase shifts $\rightarrow$ resonance masses, widths (in principle) deduced from finite-box spectrum
  - B. DeWitt, PR 103, 1565 (1956) (sphere)

- more modest goal: “ferret” out resonances from scattering states
  - must differentiate resonances from multi-hadron states
  - avoided level crossings, different volume dependences
  - know masses of decay products $\rightarrow$ placement and pattern of multi-particle states known
  - resonances show up as extra states with little volume dependence
Resonance in a toy model (I)

- O(4) non-linear $\sigma$ model (Zimmerman et al, NPB(PS) 30, 879 (1993))

$$S = -2\kappa \sum_x \sum_{\mu=1}^{4} \Phi_a(x) \Phi_a(x + \mu) + J \sum_x \Phi^4(x), \quad \sum_{a=1}^{4} \Phi_a^2(x) = 1$$
Resonance in a toy model (II)

- coupled scalar fields: (Rummukainen and Gottlieb, NPB450, 397 (1995))

\[
S = \frac{1}{2} \int d^4x \left( (\partial_\mu \phi)^2 + m_\pi^2 \phi^2 + \lambda \phi^4 + (\partial_\mu \rho)^2 + m_\pi^2 \rho^2 + \lambda_\rho \rho^4 + g \rho \phi^2 \right)
\]

\[
g = 0 \quad g = 0.008 \quad g = 0.021
\]

Figure 2. The center of mass energy levels for sectors \( \vec{P} = 0 \) (top row) and \( \vec{P} = 2\pi/L \) (bottoms) for cases A, B and C (see table 1).
Yang-Mills SU(3) Glueball Spectrum

- pure-glue mass spectrum known
  - still needs some “polishing”
  - improve scalar states
- “experimental” results in simpler world (no quarks) to help build models of gluons
- mass ratios predicted, overall scale is not
- mass gap with $1$ million bounty (Clay mathematics institute)
- glueball structure
  - constituent gluons vs flux loops?

$r_0^{-1} = 410(20) \text{ MeV}, \text{ states labeled by } J^{PC}$

Glueballs (bag model)

- qualitative agreement with bag model
  - constituent gluons are TE or TM modes in spherical cavity
  - Hartree modes with residual perturbative interactions
  - center-of-mass correction

<table>
<thead>
<tr>
<th></th>
<th>1983</th>
<th>1993</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s$</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>$B^{1/4}$</td>
<td>230 MeV</td>
<td>280 MeV</td>
</tr>
</tbody>
</table>

- recent calculation using another constituent gluon model shows qualitative agreement

Carlson, Hansson, Peterson, PRD27, 1556 (1983); J. Kuti (private communication)

Szczepaniak, Swanson, PLB577, 61 (2003)
Glueballs (flux tube model)

- disagreement with one particular string model

Isgur, Paton, PRD31, 2910 (1985)

- future comparisons:
  - with more sophisticated string models (soliton knots)
  - AdS theories, duality
**Inclusion of quark loops**

- scalar glueball results 2002
  - quark masses near strange
- still exploratory
- difficult to get adequate statistics
- light quarks problematic
- mixing, resonances
  - no correlation matrices

**References**

Unquenched masses

- unquenched analysis (Hart, Teper, PRD65, 034502 (2002))
- Wilson gauge, clover fermion action $N_f = 2, \ a \approx 0.1 \text{fm}, \ m_q \geq \frac{1}{2} m_s$
- tensor glueball mass same as pure-gauge
- scalar mass suppression: 0.85 of pure-gauge
  - not finite volume effect
  - independent of quark mass!
    - lattice artifact (another "curve ball")
  - most likely explanation: fermion action adds "adjoint piece"
- quarkonium states ignored
Excitations of static quark potential

- Gluon field in presence of static quark-antiquark pair can be excited
- Classification of states: (notation from molecular physics)
  - Magnitude of glue spin
  - Charge conjugation + parity about midpoint
  - Chirality (reflections in plane containing axis)

\[ \Lambda = 0, 1, 2, \ldots \]
\[ \eta = g \text{ (even)} \]
\[ \eta = u \text{ (odd)} \]
\[ \Pi, \Delta, \ldots \text{doubly degenerate} \]

\( \Lambda \text{ doubling} \)

\[ \Sigma, \Pi, \Delta, \ldots \]

Initial remarks

- viewpoint adopted:
  - the nature of the confining gluons is best revealed in its *excitation spectrum*
- robust feature of any bosonic string description:
  - $N\pi / R$ gaps for large quark-antiquark separations
- details of underlying string description encoded in the fine structure
- study different gauge groups, dimensionalities
- several lattice spacings, finite volume checks
- very large number of fits to principal correlators ➔ web page interface set up to facilitate scrutinizing/presenting the results
String spectrum

- spectrum expected for a non-interacting bosonic string at large $R$
  - standing waves: $m = 1, 2, 3, \ldots$ with circular polarization $\pm$
  - occupation numbers: $n_{m+}, n_{m-}$
  - energies $E$
    \[ E = E_0 + N \pi / R \]
  - string quantum number $N$
    \[ N = \sum_{m=1}^{\infty} (n_{m+} + n_{m-}) \]
  - spin projection $\Lambda$
    \[ \Lambda = \sum_{m=1}^{\infty} (n_{m+} - n_{m-}) \]
  - CP $\eta_{CP}$
    \[ \eta_{CP} = (-1)^N \]
## String spectrum \((N=1,2,3)\)

- Level orderings for \(N=1,2,3\)

<table>
<thead>
<tr>
<th>(N=0)</th>
<th>(\Sigma^+)</th>
<th>(\Pi)</th>
<th>(\Delta)</th>
<th>(\Phi)</th>
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<tbody>
<tr>
<td>(\Sigma^+_g)</td>
<td>(</td>
<td>0\rangle)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N=1)</td>
<td>(\Pi)</td>
<td>(a^+_1</td>
<td>0\rangle)</td>
<td>(a^-_1</td>
</tr>
<tr>
<td>(\Sigma^+_g)</td>
<td>(a^+_1</td>
<td>0\rangle)</td>
<td>(a^-_2</td>
<td>0\rangle)</td>
</tr>
<tr>
<td>(\Pi)</td>
<td>(a^+_2</td>
<td>0\rangle)</td>
<td>(a^-_1</td>
<td>0\rangle)</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>((a^+_1)^2</td>
<td>0\rangle)</td>
<td>((a^-_1)^2</td>
<td>0\rangle)</td>
</tr>
</tbody>
</table>

| \(N=2\) | \(\Pi\) | \(a^+_1|0\rangle\) | \(a^-_1|0\rangle\) |
| \(\Sigma^-_g\) | \(a^+_2|0\rangle\) | (\(a^+_1\)^2|0\rangle\) | (\(a^-_1\)^2|0\rangle\) |
| \(\Pi\) | \(a^+_3|0\rangle\) |
| \(\Delta\) | (\(a^+_1\)^2|0\rangle\) | (\(a^-_1\)^2|0\rangle\) |

| \(N=3\) | \(\Pi\) | \(a^+_1|0\rangle\) | \(a^-_1|0\rangle\) |
| \(\Sigma^-_g\) | (\(a^+_1\)^2|0\rangle\) | (\(a^-_1\)^2|0\rangle\) |
| \(\Pi\) | (\(a^+_2\)^2|0\rangle\) | (\(a^-_2\)^2|0\rangle\) |
| \(\Delta\) | (\(a^+_1\)^2|0\rangle\) | (\(a^-_1\)^2|0\rangle\) |

### Notes:
- \(\Sigma^+_g\) and \(\Pi\) denote the states of the system.
- \(|0\rangle\) represents the ground state.
- The superscripts \(+\) and \(-\) denote the polarization states of the string.
String spectrum \((N=4)\)

- \(N=4\) levels

| \(N = 4\) | \(\Sigma_+^{+''}\) | \(a_{2+}^+a_{2-}^-|0\rangle\) |
|-----------|----------------|---------------------------|
| \(\Sigma_+^{+''}\) | \((a_{1+}^+)^2(a_{1-}^+)^2|0\rangle\) |
| \(\Sigma_+^{+''}\) | \((a_{1+}^+a_{3-}^- + a_{1-}^-a_{3+}^+)|0\rangle\) |
| \(\Sigma_-\) | \((a_{1+}^+a_{3-}^- - a_{1-}^-a_{3+}^+)|0\rangle\) |
| \(\Pi_g^+\) | \(a_{4+}^+|0\rangle\) | \(a_{4-}^-|0\rangle\) |
| \(\Pi_g^+\) | \((a_{1+}^+)^2a_{2-}^-|0\rangle\) | \((a_{1-}^-)^2a_{2+}^+|0\rangle\) |
| \(\Pi_g^+\) | \(a_{1+}^+a_{3}^-a_{2+}^+|0\rangle\) | \(a_{1+}^+a_{1-}^-a_{2-}^-|0\rangle\) |
| \(\Delta_g^+\) | \(a_{1+}^+a_{3+}^+|0\rangle\) | \(a_{1-}^-a_{3-}^-|0\rangle\) |
| \(\Delta_g^+\) | \((a_{1+}^+)^2|0\rangle\) | \((a_{2-}^-)^2|0\rangle\) |
| \(\Delta_g^+\) | \((a_{1+}^+)^3a_{1-}^-|0\rangle\) | \(a_{1+}^+(a_{1-}^-)^3|0\rangle\) |
| \(\Phi_g\) | \((a_{1+}^+)^2a_{2+}^+|0\rangle\) | \((a_{1-}^-)^2a_{2-}^-|0\rangle\) |
| \(\Gamma_g\) | \((a_{1+}^+)^4|0\rangle\) | \((a_{1-}^-)^4|0\rangle\) |
Generalized Wilson loops

- gluonic terms extracted from generalized Wilson loops
- large set of gluonic operators $\rightarrow$ correlation matrix
- link variable smearing, blocking
- anisotropic lattice, improved actions
Ground state

\[ E_{S^+} / \sqrt{\sigma} \]

SU(3) in 4 dimensions

\( \Sigma^+ \) gauge field energy with static quark-antiquark pair
First-excited state

The graph shows the $\Pi_u$ gauge field energy with static quark-antiquark pair in SU(3) in 4 dimensions. The energy is plotted against $R \sqrt{\sigma}$, where $R$ is the distance and $\sigma$ is some parameter. The data points are color-coded for different runs (A, B, C).
First-excited state gap

\[ \Pi \] gauge field energy gap with static quark-antiquark pair

SU(3) in 4 dimensions

\[ \frac{E_{\Pi} (R) - E_{\Sigma^0} (R)}{\sqrt{\sigma}} \]

\[ R \sqrt{\sigma} \]
Three scales

- studied the energies of 16 stationary states of gluons in the presence of static quark-antiquark pair

- small quark-antiquark separations $R$
  - excitations consistent with states from multipole OPE

- crossover region $0.5\text{fm} < R < 1\text{fm}$
  - dramatic level rearrangement

- large separations $R > 1\text{fm}$
  - excitations consistent with expectations from string models

Gluon excitation gaps ($N=1,2$)

- comparison of gaps with $N\pi/R$ and Nambu-Goto
Gluon excitation gaps \((N=1,2)\)

- comparison of gaps with \(N\pi/R\) and Nambu-Goto

\[ \Delta E_{\Pi} (R) / (N\pi/R) - 1 \]

\[ \Delta E_{\Delta} (R) / (N\pi/R) - 1 \]

\[ \Xi \text{ gauge field energy gap with static quark-antiquark pair} \]

\[ \text{SU}(3) \text{ in 4 dimensions} \]
Gluon excitation gaps ($N=3$)

- comparison of gaps with $N\pi / R$ and Nambu-Goto

![Graphs showing comparisons of gauge field energy gaps with static quark-antiquark pair]
Gluon excitation gaps ($N=4$)

- comparison of gaps with $N\pi / R$ and Nambu-Goto
Possible interpretation

- small $R$
  - strong $E$ field of $q\bar{q}$-pair repels physical vacuum (dual Meissner effect) creating a bubble
  - separation of degrees of freedom
    - gluonic modes inside bubble (low lying)
    - bubble surface modes (higher lying)
- large $R$
  - bubble stretches into thin tube of flux
  - separation of degrees of freedom
    - collective motion of tube (low lying)
    - internal gluonic modes (higher lying)
  - low-lying modes described by an effective string theory ($N\pi/R$ gaps – Goldstone modes)
Heavy-quark hybrid mesons

- more amenable to theoretical treatment than light-quark hybrids
- possible treatment like diatomic molecule (Born-Oppenheimer)
  - slow heavy quarks ↔ nuclei
  - fast gluon field ↔ electrons
    (and light quarks)
- gluons provide adiabatic potentials $V_{Q\bar{Q}}(r)$
  - gluons fully relativistic, interacting
  - potentials computed in lattice simulations
- nonrelativistic quark motion described in leading order by solving Schrodinger equation for each $V_{Q\bar{Q}}(r)$
  \[
  \left\{ \frac{p^2}{2\mu} + V_{Q\bar{Q}}(r) \right\} \psi_{Q\bar{Q}}(r) = E \psi_{Q\bar{Q}}(r)
  \]
- conventional mesons from $\Sigma^+_g$; hybrids from $\Pi_u, \Sigma^-_u$, ...
Leading Born-Oppenheimer

- replace covariant derivative $\bar{D}^2$ by $\bar{\nabla}^2$ → neglects retardation
- neglect quark spin effects
- solve radial Schrödinger equation

\[
\frac{-1}{2\mu} \frac{d^2 u(r)}{dr^2} + \left( \frac{L_{qq}^2}{2\mu r^2} + V_{qq}(r) \right) u(r) = Eu(r)
\]

- angular momentum
  \[
  \bar{J} = \bar{L} + \bar{S} \quad \bar{S} = \bar{s}_q + \bar{s}_{\bar{q}} \quad \bar{L} = \bar{L}_{qq} + \bar{J}_g
  \]
- in LBO, $L$ and $S$ are good quantum numbers
- centrifugal term
  \[
  \langle \bar{L}_{qq}^2 \rangle = L(L + 1) - 2\Lambda^2 + \langle \bar{J}_g^2 \rangle \quad \langle \bar{J}_g^2 \rangle = 0 \quad \left( \Sigma^+ \right)
  \]
- $J^{PC}$ eigenstates → Wigner rotations
  \[
  |LSJM;\Lambda\eta\rangle + \varepsilon |LSJM;\Lambda\eta\rangle
  \]

  η is CP, $\varepsilon = \pm 1$ for $\Lambda \geq 1$, $\varepsilon = \pm 1$ for $\Sigma^\pm$

- LBO allowed $J^{PC}$ → $P = \varepsilon (-1)^{L+\Lambda+1}$, $C = \eta \varepsilon (-1)^{L+S+\Lambda}$
Leading Born-Oppenheimer spectrum

- results obtained (in absence of light quark loops)
- good agreement with experiment below $B\bar{B}$ threshold
- plethora of hybrid states predicted when light quark loops ignored
- but is a Born-Oppenheimer treatment valid?

LBO degeneracies:

$$\Sigma^+_g(S): \quad 0^-, 1^--$$

$$\Sigma^+_g(P): \quad 0^{++}, 1^{++}, 2^{++}, 1^{--}$$

$$\Pi_u(P): \quad 0^-, 0^{+-}, 1^{++}, 1^{--},$$

$$1^{+-}, 1^{-+}, 2^{+-}, 2^{--}$$

Charmonium LBO

- same calculation, but for charmonium
Testing LBO

- test LBO by comparison of spectrum with NRQCD simulations
  - include retardation effects, but no quark spin, no $\bar{p}^4$, no light quarks
  - allow possible mixings between adiabatic potentials
- dramatic evidence of validity of LBO
  - level splittings agree to 10% for 2 conventional mesons, 4 hybrids

$$H_1, H_1' = 1^{--}, 0^{++}, 1^{--}, 2^{++}$$

$$H_2 = 1^{++}, 0^{+-}, 1^{++, 2^{--}}$$

$$H_3 = 0^{++}, 1^{--}$$

lowest hybrid 1.49(2)(5) GeV above 1S
Light quark spoiler?

- spoil B.O.? → unknown
- light quarks change $V_{qar{q}}(r)$
  - small corrections at small $r$
    - fixes low-lying spectrum
  - large changes for $r>1$ fm
    → fission into $(Qar{q})(Qar{q})$
- states with diameters over 1 fm
  - most likely cannot exist as observable resonances
- dense spectrum of states from pure glue potentials will not be realized
  - survival of a few states conceivable given results from Bali et al.
- discrepancy with experiment above $B\bar{B}$
  - most likely due to light quark effects
charge from Nathan Isgur to use Monte Carlo method to extract the spectrum of baryon resonances (Hall B at JLab)

formed the **Lattice Hadron Physics Collaboration (LHPC)** in 2000

acquired funding through DOE SciDAC to build large computing cluster at JLab (also at Fermilab and Brookhaven), develop software

LHPC has several broad goals

- compute QCD spectrum (baryons, mesons,...)
- hadron structure (form factors, structure functions,...)
- hadron-hadron interactions

current members of spectroscopy effort:

- Subhasish Basak, Robert Edwards, George Fleming, Jimmy Juge, Adam Lichtl, CM, David Richards, Ikuro Sato, Steve Wallace
LHPC spectroscopy efforts

- extracting spectrum of resonances is big challenge!!
  - need sets of extended operators (correlator matrices)
  - multi-hadron operators needed too
  - deduce resonances from finite-box energies
  - anisotropic lattices \( (a_t < a_s) \)
  - inclusion of light-quark loops at realistically light quark mass
- long-term project
Operator design issues

- must facilitate spin identification
  - shun the usual method of operator construction which relies on cumbersome continuum space-time constructions
  - focus on constructing operators which transform irreducibly under the symmetries of the lattice
- one eye on maximizing overlaps with states of interest
- other eye on minimizing number of quark-propagator sources
- use building blocks useful for baryons, mesons, multi-hadron operators
Three stage approach (hep-lat/0506029)

- concentrate on baryons at rest (zero momentum)
- operators classified according to the irreps of $O_h$
  $$G_{1g}, G_{1u}, G_{2g}, G_{2u}, H_g, H_u$$
- (1) basic building blocks: smeared, covariant-displaced quark fields
  $$\left(\tilde{D}_j^{(p)}\tilde{\psi}(x)\right)_{Aa\alpha} \quad p\text{- link displacement } (j = 0, \pm 1, \pm 2, \pm 3)$$
- (2) construct elemental operators (translationally invariant)
  $$B^F(x) = \phi^F_{ABC} \varepsilon_{abc} \left(\tilde{D}_i^{(p)}\tilde{\psi}(x)\right)_{Aa\alpha} \left(\tilde{D}_j^{(p)}\tilde{\psi}(x)\right)_{Bb\beta} \left(\tilde{D}_k^{(p)}\tilde{\psi}(x)\right)_{Cc\gamma}$$
  - flavor structure from isospin, color structure from gauge invariance
- (3) group-theoretical projections onto irreps of $O_h$
  $$B^{\Lambda\bar{\Lambda}F}_i(t) = \frac{d_\Lambda}{g_{Q_h^D \cdot R = Q_h^D}} \sum_R D^{(\Lambda)}_\Lambda(R)^* U_R B^F_i(t) U_R^+$$
  - wrote Grassmann package in Maple to do these calculations
Three-quark elemental operators

- three-quark operator

\[ \Phi_{ABC}^{\alpha\beta\gamma,ijk}(t) = \sum_{\lambda} \varepsilon_{abc} (\tilde{D}^{(p)}_{\lambda}(\bar{\psi}(\bar{x},t))_{\alpha\lambda}) A (\tilde{D}^{(p)}_{\lambda}(\bar{\psi}(\bar{x},t))_{\beta\lambda}) B (\tilde{D}^{(p)}_{\lambda}(\bar{\psi}(\bar{x},t))_{\gamma\lambda}) C \]

- covariant displacements

\[ \tilde{D}^{(p)}_{j}(x,x') = \tilde{U}_{j}(x) \tilde{U}_{j}(x + \hat{j}) \ldots \tilde{U}_{j}(x + (p - 1) \hat{j}) \delta_{x',x + p\hat{j}} \quad (j = \pm 1, \pm 2, \pm 3) \]

\[ \tilde{D}^{(p)}_{0}(x,x') = \delta_{x',x} \]

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Operator</th>
</tr>
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<tr>
<td>( \Delta^{++} )</td>
<td>( \Phi_{uuu}^{\alpha\beta\gamma,ijk} )</td>
</tr>
<tr>
<td>( \Sigma^{+} )</td>
<td>( \Phi_{uus}^{\alpha\beta\gamma,ijk} )</td>
</tr>
<tr>
<td>( N^{+} )</td>
<td>( \Phi_{uud}^{\alpha\beta\gamma,ijk} - \Phi_{duu}^{\alpha\beta\gamma,ijk} )</td>
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<tr>
<td>( \Xi^{0} )</td>
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<td>( \Lambda^{0} )</td>
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<tr>
<td>( \Omega^{-} )</td>
<td>( \Phi_{sss}^{\alpha\beta\gamma,ijk} )</td>
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</table>
Incorporating orbital and radial structure

- displacements of different lengths build up radial structure
- displacements in different directions build up orbital structure

operator design minimizes number of sources for quark propagators

useful for mesons, tetraquarks, pentaquarks even!

- can even incorporate **hybrid mesons** operator (in progress)
Enumerating the three-quark operators

- lots of operators (too many!)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta^{++}$, $\Omega^-$</th>
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Spin identification and other remarks

- Spin identification possible by pattern matching

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Total numbers of operators assuming two different displacement lengths

- Total numbers of operators is huge $\rightarrow$ uncharted territory
- Ultimately must face two-hadron scattering states
Single-site operators

- choose Dirac-Pauli convention for $\gamma$-matrices

- 20 independent single-site $\Delta^{++}$ elemental operators:
  \[ \Delta_{\alpha\beta\gamma} = \varepsilon_{abc} \bar{u}_{a\alpha} \bar{u}_{b\beta} \bar{u}_{c\gamma}, \quad (\alpha \leq \beta \leq \gamma) \]

- 20 independent single-site $N^+$ elemental operators:
  \[ N_{\alpha\beta\gamma} = \varepsilon^{abc} (\bar{u}_{a\alpha} \bar{u}_{b\beta} \bar{d}_{c\gamma} - \bar{d}_{a\alpha} \bar{u}_{b\beta} \bar{u}_{c\gamma}), \quad (\alpha \leq \beta, \alpha < \gamma) \]

- 40 independent single-site $\Sigma^+$ elemental operators:
  \[ \Sigma_{\alpha\beta\gamma} = \varepsilon_{abc} \bar{u}_{a\alpha} \bar{u}_{b\beta} \bar{s}_{c\gamma} \quad (\alpha \leq \beta) \]

- 24 independent single-site $\Lambda^0$ elemental operators:
  \[ \Lambda_{\alpha\beta\gamma} = \varepsilon_{abc} \left( \bar{u}_{a\alpha} \bar{d}_{b\beta} \bar{s}_{c\gamma} - \bar{d}_{a\alpha} \bar{u}_{b\beta} \bar{s}_{c\gamma} \right) \quad (\alpha < \beta) \]
### $\Delta^{++}$ single-site operators

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<th>Irrep</th>
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## Single-site $N+$ operators

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<tr>
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<td>4</td>
<td>$-\sqrt{3} N_{113}$</td>
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</tbody>
</table>
Current status and next step

- Development of software to carry out the baryon computations has been completed and thoroughly tested (at long last!)
  - gauge-invariant three-quark propagators as intermediate step
  - baryon correlators are superpositions of $qqq$-propagator components $\rightarrow$ superposition coefficients precalculated
  - source-sink rotations to minimize source orientations
- Next step: smearing optimization and operator pruning
  - optimize link-variable and quark-field smearings
  - remove dynamically redundant operators
  - remove ineffectual operators
  - low statistics runs needed
  - monitor progress at http://enrico.phys.cmu.edu
Quark- and gauge-field smearing

- smeared quark and gluon fields fields → dramatically reduced coupling with short wavelength modes

- **link-variable** smearing (stout links PRD69, 054501 (2004))
  - define $C_\mu(x) = \sum_{\pm(v \neq \mu)} \rho_{\mu v} U_v(x) U_\mu(x + \hat{v}) U^+_v(x + \hat{\mu})$
  - spatially isotropic $\rho_{jk} = \rho$, $\rho_{4k} = \rho_{k4} = 0$
  - exponentiate traceless Hermitian matrix
    $$\Omega_\mu = C_\mu U^+_\mu, \quad Q_\mu = \frac{i}{2} (\Omega^+_\mu - \Omega_\mu) - \frac{i}{2N} \text{Tr}((\Omega^+_\mu - \Omega_\mu))$$
  - iterate
    $$U^{(n+1)}_\mu = \exp(iQ^{(n)}_\mu) U^{(n)}_\mu$$
    $$U_\mu \rightarrow U^{(1)}_\mu \rightarrow \cdots \rightarrow U^{(n)}_\mu \equiv \tilde{U}_\mu$$

- **quark-field** smearing (covariant Laplacian uses smeared gauge field)
  $$\bar{\psi}(x) = \left(1 + \frac{\sigma_s}{4n_\sigma} \tilde{\Delta}^2\right)^{n_\sigma} \psi(x)$$
Importance of smearing

- Nucleon G1g channel
- Effective masses of 3 selected operators
- Noise reduction from link variable smearing, especially for displaced operators
- Quark-field smearing reduces couplings to high-lying states
  \[ \sigma_s = 4.0, \quad n_{\sigma} = 32 \]
  \[ n_{\rho}\rho = 2.5, \quad n_{\rho} = 16 \]
- Effect on excited states still to be studied
Tuning the smearing

- The effective mass at $t = 4a_t$ for three specific nucleon operators for different quark-field smearings (link smearing same as last slide)
Operator plethora (G1g Nucleon)

- Single Site Operator 0
- Single Site Operator 1
- Three-Link Singly Displaced Operator 0
- Three-Link Singly Displaced Operator 3
- Single Site Operator 2
- Three-Link Singly Displaced Operator 1
- Three-Link Singly Displaced Operator 5
- Three-Link Singly Displaced Operator 6
- Three-Link Singly Displaced Operator 7
- Three-Link Singly Displaced Operator 8
G1g nucleon operators
G1g nucleon operators

Three-Link Doubly Displaced L Operator 9

Three-Link Doubly Displaced L Operator 10

Three-Link Doubly Displaced L Operator 11

Three-Link Doubly Displaced L Operator 12

Three-Link Triply Displaced T Operator 43

Three-Link Triply Displaced T Operator 44

Three-Link Triply Displaced T Operator 45

Three-Link Triply Displaced T Operator 46
G1g nucleon operators

Three-Link Triply Displaced T Operator 11

Three-Link Triply Displaced T Operator 12

Three-Link Triply Displaced T Operator 13

Three-Link Triply Displaced T Operator 41

Three-Link Triply Displaced T Operator 42

Three-Link Triply Displaced T Operator 43

Three-Link Triply Displaced T Operator 44

Three-Link Triply Displaced T Operator 45

Three-Link Triply Displaced T Operator 46
G2g nucleon operators
Hu nucleon operators

Single Site Operator 0

Three-Link Singly Displaced Operator 0

Three-Link Singly Displaced Operator 1

Three-Link Singly Displaced Operator 2

Three-Link Singly Displaced Operator 3

Three-Link Singly Displaced Operator 4

Three-Link Triply Displaced T Operator 35

Three-Link Triply Displaced T Operator 36

Three-Link Triply Displaced T Operator 37

Three-Link Triply Displaced T Operator 38

Three-Link Triply Displaced T Operator 39

Three-Link Triply Displaced T Operator 40
Principal effective masses

- principal effective masses for small set of 10 operators

\[ G_{1g} \text{ on left, other irreps on right.} \]
Future work

- three-quark operator pruning
  - principal effective masses
- include mesons
- include multihadron operators
  - stochastic all-to-all propagators with dilution, low eigenvectors
  - will also facilitate disconnected diagrams
- our goal is to have the operator technology ready to go when unquenched simulations near realistically-light quark masses become possible
  - continuing efforts by lattice community, including other members of LHPC
Summary

- described explorations of QCD spectrum using lattice Monte Carlo
- glueball mass spectrum in pure gauge theory
- stationary states of gluons in presence of static quark-antiquark pair as a function of separation $R$
  - unearthed the effective QCD string for $R > 1$ fm for the first time
  - tantalizing fine structure revealed effective string action clues
- heavy-quark hybrid mesons (Born-Oppenheimer treatment)
- outlined ongoing efforts of LHPC to extracting baryon spectrum with large sets of extended operators
  - applications to mesons (and hybrids), tetraquark, pentaquark
  - emphasized need for correlation matrices to extract spectrum
  - spin identification must be addressed
  - multi-hadron operators will become important
  - very challenging calculations